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QCD vacuum tensor susceptibility and properties of transversely polarized mesons

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Abstract. We re-estimate the tensor susceptibility of the QCD vacuum, χ, and to this end, we re-estimate the tensor coupling constants for the transversely polarized ρ-, ρ'- and b_1-mesons. The origin of the susceptibility is analyzed using (anti-)duality between the ρ- and b_1- channels in the 2-point correlator of the tensor currents. We conclude that the origin of the differences in ρ- and b_1-meson masses and tensor couplings is the (anti-)duality breakdown in QCD due to the 4-quark condensate. We confirm the results of Govaerts et al. for the 2-point correlator of the tensor currents and disagree with Belyaev and Oganesyan on both the OPE expansion and the value of the QCD vacuum tensor susceptibility. Using our value for the latter we determine new estimations of the nucleon tensor charges related to the first moment of the transverse structure function h_1 of a nucleon.

1 Introduction

In this paper, we investigate the low-energy properties of the lightest transversely polarized mesons with quantum numbers $J^{PC} = 1^{--}(\rho,\rho'), 1^{+-}(b_1)$ in the framework of QCD sum rules (SRs) with non-local condensates (NLCs) as well as with the standard ones. This work was started in [3], where the "mixed parity" NLC SR for the light-cone distribution (LCD) amplitudes of both the ρ- and the b_1-meson was constructed. It was concluded that to obtain a reliable result we should reduce model uncertainties due to the non-local gluon contribution into the SR for LCD. Different SRs for each P-parity could be preferable for this purpose. As a first step, to obtain the twist 2 meson LCD, we concentrate on the meson static properties using the "pure parity" NLC SR for each meson separately:

(1) we re-estimate the tensor coupling constants f_m^{T} for transversely polarized $\rho_\perp(770)$ (1^{--}) and $b_{1\perp}(1235)$ (1^{+-})-mesons and estimate a (new) one for $\rho'_\perp(1465)$-meson [4];

(2) we correct the previous consideration by Belyaev and Oganesyan (B&O) [2] and provide a new estimation for the vacuum tensor susceptibility (VTS) introduced in [5, 6].

The static characteristics, the coupling constants f_m^{T} and "continuum thresholds" s_m (parameters of the phenomenological models for the spectral densities) of the lightest transversely polarized mesons in the channels with $J^{PC} = 1^{--}$ and 1^{+-} are tightly connected with the value of the VTS. Namely, the difference of the meson properties in these channels fixes the non-zero value of the VTS: in a hypothetical model of Nature, e.g., where the properties of these mesons are the same, VTS is identically equal to zero. For the reason that these meson constants should appear in VTS in the form of a difference, one has to define them more precisely and in the framework of a unified approach.

The approach of the NLC SRs was successfully applied for the determination of meson dynamic characteristics (LCD amplitudes, form factors; see, e.g., [3,7] and references therein). For the readers' convenience some important features of the approach should be recalled. The original tool of NLC SR is the non-local objects like $M(z^2) = \langle\bar{q}(0)E(0,z)q(z)\rangle$[1], rather than $\langle\bar{q}(0)q(0)\rangle$. The NLC $M(z^2)$ can be expanded over the standard (local) condensates, $\langle\bar{q}(0)q(0)\rangle$, $\langle\bar{q}(0)\nabla^2 q(0)\rangle$, "higher dimensions". So, one can come back to the standard SR by truncating this series. But, in virtue of the truncation, one loses an important physical property of the non-perturbative vacuum – the possibility of vacuum quarks (gluons) flowing through the vacuum with non-zero momentum $k_{q(g)} \neq 0$. The parameter $\langle k_q^2\rangle$ fixing the average virtuality of the vacuum quarks was estimated from the mixed condensate of dimension 5, $\langle k_q^2\rangle = \lambda_q^2 \approx 0.4 - 0.5\,\mathrm{GeV}^2$ [8] (see Appendix A, (A.6)). This value is of the order of the hadronic scale, $m_\rho^2 \approx 0.6\,\mathrm{GeV}^2$; therefore it should be taken into account in QCD SR for light hadrons. Since neither QCD vacuum theory exists, nor the higher dimension condensates are estimated, it is clear that merely the models of the NLC can be suggested. Here we apply the simplest ansatz [7], which takes into account only the main effect $\langle k_q^2\rangle \neq 0$ and

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[1] Here $E(0,z) = P\exp\left(\mathrm{i}\int_0^z \mathrm{d}t_\mu A_\mu^a(t)\tau_a\right)$ is the Schwinger phase factor required for gauge invariance

leads to Gaussian decay for the NLC, while the quantity $1/\lambda_q$ reveals itself as the length of the quark–gluon correlation in the QCD vacuum [7]. It is important to note that the non-local character of the quark condensate has recently been confirmed in direct lattice calculations [9, 10]. The latter measurement in [10] confirms the validity of the Gaussian ansatz, as well as the value of the parameter λ_q^2.

The NLC approach can improve the stability and accuracy of the SRs even for determining the coupling constants where the NLC effect is of the order of the radiative correction contribution. Note that the effect is not sensitive to the details of the specific NLC ansatz. Therefore, we revise the values of these static meson characteristics in pure parity NLC SRs, though different estimations for these quantities can be found in the literature [11,2,3] obtained in different ways. For comparison, we also calculate all these quantities in the standard way which corresponds to processing our NLC SR in the limit $\lambda_q^2 \to 0$.

The above-mentioned difference of meson properties is due to the specific four-quark condensate contribution to the "theoretical part" of the SRs. This contribution is invariant under the duality transformation in contrast to all other condensate contributions which change sign under the same transformation. This peculiarity of the four-quark condensate contribution will be considered in detail.

The plan of presentation is the following: first, we consider the QCD SR approach to investigate the 4-rank tensor 2-point correlator for transversely polarized ρ-, ρ'- and b_1-mesons. Then we define the duality transformation and draw consequences from it for the constructed SRs. Finally, we derive a new estimation for the QCD VTS and nucleon tensor charges and discuss what is wrong in the consideration of [2].

2 Tensor couplings for ($J^{PC} = 1^{--}, 1^{+-}$)-mesons

We start with the 2-point correlator of tensor currents, $J^{\mu\nu}(x) = \bar{u}(x)\sigma^{\mu\nu}d(x)$,

$$\Pi^{\mu\nu;\alpha\beta}(q) = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T[J^{\mu\nu +}(x)J^{\alpha\beta}(0)]|0\rangle. \quad (1)$$

(Note here that due to isospin symmetry this is the same correlator as was studied in [2].) This correlator can be decomposed in invariant form factors $\Pi_\pm$ [1,11],

$$\Pi^{\mu\nu;\alpha\beta}(q) = \Pi_-(q^2)P_1^{\mu\nu;\alpha\beta} + \Pi_+(q^2)P_2^{\mu\nu;\alpha\beta}, \quad (2)$$

where the projectors $P_{1,2}$ are defined by the expressions

$$P_1^{\mu\nu;\alpha\beta} \equiv \frac{1}{2q^2}\left[g^{\mu\alpha}q^\nu q^\beta - g^{\nu\alpha}q^\mu q^\beta - g^{\mu\beta}q^\nu q^\alpha + g^{\nu\beta}q^\mu q^\alpha\right]; \quad (3)$$

$$P_2^{\mu\nu;\alpha\beta} \equiv \frac{1}{2}\left[g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha}\right] - P_1^{\mu\nu;\alpha\beta}, \quad (4)$$

which obey the projector-type relations

$$\begin{aligned}(P_i\cdot P_j)^{\mu\nu;\alpha\beta} &\equiv P_i^{\mu\nu;\sigma\tau}P_j^{\sigma\tau;\alpha\beta} \\ &= \delta_{ij}P_i^{\mu\nu;\alpha\beta} \text{ (no sum over } i), \\ P_i^{\mu\nu;\mu\nu} &= 3.\end{aligned} \quad (5)$$

Then for the form factors $\Pi_\pm(q^2)$ it is possible to use dispersion representations of the form

$$\Pi_\pm(q^2) = \frac{1}{\pi}\int_0^\infty \frac{\rho_\pm(s)\mathrm{d}s}{s-q^2} + \text{subtractions}, \quad (6)$$

which after the Borel transformation (with Borel parameter M^2) become

$$\Pi_\pm(q^2) \to B\Pi_\pm(M^2) = \frac{1}{\pi M^2}\int_0^\infty \rho_\pm(s)\mathrm{e}^{-s/M^2}\mathrm{d}s. \quad (7)$$

A phenomenological model for the spectral density $\rho^{\mathrm{phen}}(s)$ is usually taken in the form of "lowest resonances + continuum":

$$\begin{aligned}\rho_\pm^{\mathrm{phen}}(s) &= \pm 2\pi\left|f_m^{\mathrm{T}}\right|^2 s\cdot\delta(s-m_m^2) \\ &+\rho_\pm^{\mathrm{pert}}(s)\theta(s-s_\pm),\end{aligned} \quad (8)$$

where f_m^{T} and m_m are the constants and masses of the lowest meson resonances, $m = \rho, \rho', b_1$, contributing to the correlator of interest, and the $\rho_\pm^{\mathrm{pert}}(s)$ are the corresponding spectral densities of the perturbative contributions to the correlators $\Pi_\pm(q^2)$. The coupling constants f_m^{T} are defined via the parameterization of the unit helicity ($|\lambda| = 1$) states of the ρ-, ρ'- and b_1-mesons

$$\begin{aligned}&\langle 0\,|\bar{u}(x)\sigma_{\mu\nu}d(x)|\,\rho^+(p,\lambda)(\rho'^+)\rangle \\ &\quad = \mathrm{i}f_{\rho,\rho'}^{\mathrm{T}}\left(\varepsilon_\mu(p,\lambda)p_\nu - \varepsilon_\nu(p,\lambda)p_\mu\right);\end{aligned} \quad (9)$$

$$\langle 0\,|\bar{u}(x)\sigma_{\mu\nu}d(x)|\,b_1^+(p,\lambda)\rangle = f_{b_1}^{\mathrm{T}}\epsilon_{\mu\nu\alpha\beta}\varepsilon^\alpha(p,\lambda)p_\beta; \quad (10)$$

here $\varepsilon^\mu(p,\lambda)$ is the polarization vector of a meson with momentum p and helicity λ. To construct the SRs, one should calculate the OPE of the correlators $\Pi_\pm(M^2)$

$$\begin{aligned}B\Pi_\pm(M^2) &= \frac{1}{\pi M^2}\int_0^\infty \rho_\pm^{\mathrm{pert}}(s)\mathrm{e}^{-s/M^2}\mathrm{d}s + \frac{a_\pm}{M^2}\left\langle\frac{\alpha_{\mathrm{s}}}{\pi}G^2\right\rangle \\ &+ \frac{b_\pm}{M^4}\pi\langle\sqrt{\alpha_{\mathrm{s}}}\bar{q}q\rangle^2.\end{aligned} \quad (11)$$

We perform these calculations in the approach of the QCD SRs with NLCs (see [3]), where the coefficients $a_\pm, b_\pm$ become functions $a_\pm(M^2), b_\pm(M^2)$ of the Borel parameter M^2 which tend to their standard values for large M^2, $M^2 \gg \lambda_q^2$, e.g., $b_\pm = \lim_{(\lambda_q^2/M^2)\to 0} b_\pm(M^2)$. The functions $a_\pm(M^2), b_\pm(M^2)$ can be considered as accumulating an infinite subset of the standard condensate $\left(\lambda_q^2/M^2\right)^j$-contributions [7] in OPE. All needed NLC expressions are given in Appendix A, while the standard coefficients $a_\pm, b_\pm$, corresponding to the limit $\lambda_q^2/M^2 \to 0$, are explicitly written below. Their values are in full agreement with the preceding calculations performed in [1,11]

$$\frac{1}{(\pm 2)}\rho^{\rm pert}_{\pm}(s) = \rho^{\rm pert}_{0}(s) \equiv \frac{s}{8\pi}\left[1+\frac{\alpha_{\rm s}(\mu^2)}{\pi} \times\left(\frac{7}{9}+\frac{2}{3}\log\frac{s}{\mu^2}\right)\right]; \tag{12}$$

$$\frac{1}{(\pm 2)}a_{\pm} = \frac{1}{24}; \tag{13}$$

$$\frac{1}{(-2)}b_{-} = \frac{-16+80+144}{81} = \frac{208}{81}; \tag{14}$$

$$\frac{1}{(+2)}b_{+} = \frac{-16+80-144}{81} = \frac{-80}{81}. \tag{15}$$

Here μ is the renormalization scale ($\mu^2 \simeq 1\,\mathrm{GeV}^2$) and the coefficients listed in the central parts of the last two lines correspond to the vector $\langle\bar{q}\gamma_m q\rangle$, quark–gluon–quark $\langle\bar{q}G_{\mu\nu}q\rangle$ and the four-quark $\langle\bar{q}q\bar{q}q\rangle$ vacuum condensate contributions (see details in Appendix A, [7]). We write down these coefficients explicitly in order to reveal the discrepancy between our results and those obtained by B&O [2], who found, instead, in the last line

$$\frac{-16-48-144}{81} = \frac{-208}{81},$$

a result larger than ours by a factor of 2.6. We conclude that in [2] there is a wrong contribution due to the quark–gluon–quark vacuum condensate.

Collecting all parts, (7), (8) and (11), together, one obtains the following SRs:

$$\left|f^{\rm T}_{\rho}\right|^2 m_{\rho}^2 {\rm e}^{-m_{\rho}^2/M^2} + (\rho\to\rho') = \frac{1}{\pi}\int_0^{s_\rho}\rho^{\rm pert}_0(s){\rm e}^{-s/M^2}{\rm d}s - \frac{a_-}{2}\left\langle\frac{\alpha_{\rm s}}{\pi}G^2\right\rangle - \frac{b_-(M^2)}{2M^2}\pi\langle\sqrt{\alpha_{\rm s}}\bar{q}q\rangle^2; \tag{16}$$

$$\left|f^{\rm T}_{b_1}\right|^2 m_{b_1}^2 {\rm e}^{-m_{b_1}^2/M^2} = \frac{1}{\pi}\int_0^{s_{b_1}}\rho^{\rm pert}_0(s){\rm e}^{-s/M^2}{\rm d}s + \frac{a_+}{2}\left\langle\frac{\alpha_{\rm s}}{\pi}G^2\right\rangle + \frac{b_+(M^2)}{2M^2}\pi\langle\sqrt{\alpha_{\rm s}}\bar{q}q\rangle^2. \tag{17}$$

The role of the NLC, concentrated in $a_{\pm}, b_{\pm}(M^2)$, is important here, i.e., at $M^2 = 0.6\,\mathrm{GeV}^2$ the total condensate contribution in the SR reduces twice in comparison with the standard (local) one. In accordance with QCD SR practice, the processing of these NLC SRs are performed within the validity window $M^2_- \le M^2 \le M^2_+$ (see details in [12,3]). These windows are determined by two conditions: the lower bound M^2_- by demanding that the relative value of the $\langle GG\rangle$- and $\langle\bar{q}q\rangle$-contributions to the OPE series should not be larger than 30%, the upper bound M^2_+ by requiring that a relative contribution of the higher states in the phenomenological part of SR should not be larger than 30%. The processing with the standard values of the vacuum condensates (see Appendix A) gives the constants

$$f^{\rm T}_{\rho} = 0.157\pm 0.005\,\mathrm{GeV},\ f^{\rm T}_{\rho'} = 0.140\pm 0.005\,\mathrm{GeV},\quad s^{\rm T}_{\rho,\rho'} = 2.8\,\mathrm{GeV}^2; \tag{18}$$

$$f^{\rm T}_{b_1} = 0.184\pm 0.005\,\mathrm{GeV}, \quad s^{\rm T}_{b_1} = 2.87\,\mathrm{GeV}^2, \tag{19}$$

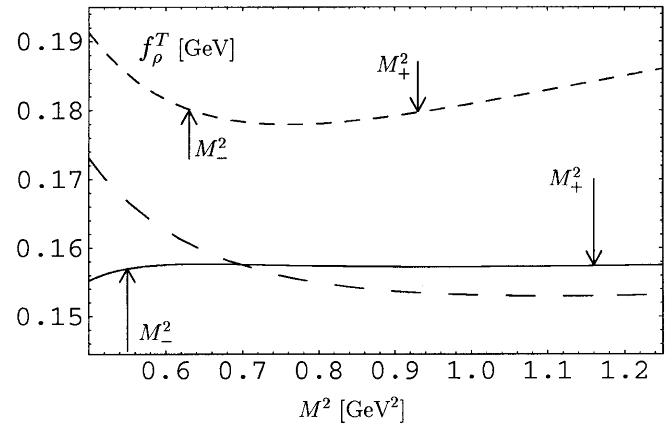

Fig. 1. The curves of $f^{\rm T}_{\rho}$ in M^2; the solid line corresponds to the NLC SR with the ρ'-meson taken into account, the long arrows show its validity window; the short-dashed line corresponds to the standard SR without ρ'-meson, the small arrows show the reduced validity window for this case; the dashed line corresponds to the B&B analyses

which are presented at the normalization point $\mu^2 = 1\,\mathrm{GeV}^2$. Very stable curves in wide validity windows have been obtained for all of these quantities.

The processing of the “local” version (at $\lambda_q^2 \to 0$) of the SRs (16)–(17) leads to the values[2]

$$f^{\rm T}_{\rho} = 0.179(0.170)\pm 0.007\,\mathrm{GeV},\ f^{\rm T}_{\rho'} \sim 0\,\mathrm{GeV},\quad s^{\rm T}_{\rho,\rho'} = 2.1\,\mathrm{GeV}^2; \tag{20}$$

$$f^{\rm T}_{b_1} = 0.191(0.178)\pm 0.009\,\mathrm{GeV}, \quad s^{\rm T}_{b_1} = 3.2\,\mathrm{GeV}^2, \tag{21}$$

which is an accuracy which looks worse. Really, the curve corresponding to $f^{\rm T}_{\rho}$ in M^2 is shown in Fig. 1 (solid line) in comparison with the result of the standard approach without ρ'-meson (short-dashed line). For the first case, the validity window expands in the whole region (0.55–1.20) GeV^2 (long arrows) while for the latter case it shrinks twice to the region denoted in figure by the small arrows M^2_- and M^2_+. Note that the standard SR “pushes out” the ρ' meson and does not allow one to obtain its parameters, while the NLC SR is sensitive to this meson and even allows to determine its mass [3]. We demonstrate on the same figure the curve for $f^{\rm T}_{\rho}$ (dashed line), obtained in [11] by Ball and Braun (B&B) in the framework of the standard approach, the same small arrows denoting its real “working window”. Note here that processing the B&B SR just in this thin working window results in a curve very similar in shape to the upper short-dashed one with the average value $f^{\rm T}_{\rho}(1\mathrm{GeV}^2) = 0.171\,\mathrm{GeV}$. Note

[2] To provide a clear comparison with the results of B&O, who do not take into account ρ'-meson contribution, the condensate non-locality and $\alpha_{\rm s}$-corrections in the perturbative spectral density, we write down the results of processing our SRs in the same approximation in parentheses

Table 1. Estimates for the coupling constants $f^{\mathrm{T}}(1\mathrm{GeV}^2)$ of transversely polarized $\rho(770)$-, $\rho'(1465)$- and $b_1(1235)$-mesons based on processing the QCD SRs in different approaches

	"Pure parity" SR based on $\Pi_{\mp}$ ($-$ for ρ, $+$ for b_1)			"Mixed parity" SR based on $(\Pi_- - \Pi_+)/q^2$	
Source	Here	B&B [11]	B&O [2]	Here[a]	B&B [11]
f^{T}_{ρ} [MeV]	157(5)	160(10)	–	166(6)	163(5)
$f^{\mathrm{T}}_{\rho'}$ [MeV]	140(5)	–	–	–	–
s_ρ $\left[\mathrm{GeV}^2\right]$	2.8	1.5	–	1.5	2.1
$f^{\mathrm{T}}_{b_1}$ [MeV]	184(5)	180(10)	178(10)	179(7)	180^{fixed}
s_{b_1} $\left[\mathrm{GeV}^2\right]$	2.87	2.7	3.0	2.93	2.1

[a] The estimates presented in this column have been obtained by processing the "mixed parity" SR established in [3]. We improve the model for the phenomenological spectral density using the features of phenomenological spectral densities of "pure parity" SRs

that recently performed lattice estimates [13][3] give $f^{\mathrm{T}}_{\rho\ \mathrm{Latt}}$ $(4\,\mathrm{GeV}^2) = 0.165(11)\,\mathrm{GeV}$, which approximately agrees with both the NLC (18) and the "standard" values of (20). So we can conclude that our improved SRs (16) and (17) are really justified and produce reliable and stable results. All the results obtained by processing "pure parity", (16) and (17), and "mixed parity" NLC SR [3] are collected in Table 1, in comparison with the previous results in [11,2].

It is interesting to note that in spite of the discrepancy in the OPE coefficients, the authors of [2] obtain for $f^{\mathrm{T}}_{b_1}$ a value of $178 \pm 10\,\mathrm{MeV}$ which is quite close to the value found by B&B [11]: $180 \pm 10\,\mathrm{MeV}$. This compensation effect occurs due to the fact that both groups of authors used different sets of condensate input parameters in the SR and this resulted in approximately the same overall contributions of the quark condensate: B&B had $((1/2)b_+)\,\pi\alpha_{\mathrm{s}}\langle\bar{q}q\rangle^2 = -4.2210^{-4}\,\mathrm{GeV}^6$; and B&O, $((1/2)b_+)\,\pi\alpha_{\mathrm{s}}\langle\bar{q}q\rangle^2 = -4.9210^{-4}\mathrm{GeV}^6$, see Appendix B.

3 Duality and its breakdown

Let us consider now an operator $\hat{D}$ transforming any rank-4 tensor $T^{\mu\nu;\alpha\beta}$ to another rank-4 tensor $T_D^{\mu\nu;\alpha\beta} = (\hat{D}T)^{\mu\nu;\alpha\beta}$ with

$$D^{\mu\nu;\alpha\beta}_{\mu'\nu';\alpha'\beta'} = \frac{-1}{4}\epsilon^{\mu\nu}{}_{\mu'\nu'}\epsilon_{\alpha'\beta'}{}^{\alpha\beta} \text{ and } \hat{D}^2 = 1. \tag{22}$$

Our projectors $P_1^{\mu\nu;\alpha\beta}$ and $P_2^{\mu\nu;\alpha\beta}$ under the action of this operator transform into each other

$$\left(\hat{D}P_1\right)^{\mu\nu;\alpha\beta} = P_2^{\mu\nu;\alpha\beta};\ \left(\hat{D}P_2\right)^{\mu\nu;\alpha\beta} = P_1^{\mu\nu;\alpha\beta}, \tag{23}$$

[3] We are indebted to D. Becirevic, who informed us about these interesting papers, containing lattice estimates of masses and coupling constants of mesons

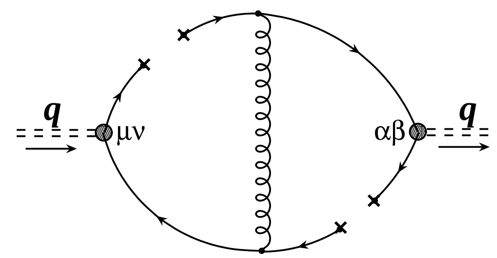

Fig. 2. Diagram with insertion of four-quark condensate

whereas the correlator $\Pi^{\mu\nu;\alpha\beta}(q)$ transforms into the correlator of dual tensor currents, $J_5^{\mu\nu}(x) = \bar{u}(x)\sigma^{\mu\nu}\gamma_5 d(x)$,

$$(\hat{D}\Pi)^{\mu\nu;\alpha\beta}(q) = \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T[J_5^{\mu\nu+}(x)J_5^{\alpha\beta}(0)]|0\rangle. \tag{24}$$

Now one is faced with a question: How are $\Pi^{\mu\nu;\alpha\beta}(q)$ and $(\hat{D}\Pi)^{\mu\nu;\alpha\beta}(q)$ connected?

In perturbative QCD with massless fermions, taking into account the standard anticommutations, one easily arrives at

$$(\hat{D}\Pi)^{\mu\nu;\alpha\beta}_{\mathrm{pert}}(q) = -\Pi^{\mu\nu;\alpha\beta}_{\mathrm{pert}}(q), \tag{25}$$

from which it follows that $\Pi^{\mu\nu;\alpha\beta}_{\mathrm{pert}}(q)$ is anti-self-dual. The same (anti-dual) character is inherent in the phenomenological models, see (8).

The same reasoning is valid almost for all OPE diagrams; those with a gluon condensate, with a vector quark condensate, and with a quark–gluon–quark condensate. Only the diagram with four-quark scalar condensates is different (see Fig. 2): in that case there are two γ-matrices on one line between two external vertices (one from the fermion propagator and one from the quark–gluon vertex) because the scalar condensate cancels one γ-matrix. Thus, we realize that the OPE contribution involves two parts, one being anti-self-dual (**ASD**) and the other one self-dual (**SD**):

$$\Pi^{\mu\nu;\alpha\beta}_{\mathrm{OPE}}(q) = \mathbf{ASD}^{\mu\nu;\alpha\beta}(q) + \mathbf{SD}^{\mu\nu;\alpha\beta}(q), \tag{26}$$

$$(-\hat{D}\mathbf{ASD})^{\mu\nu;\alpha\beta}(q) = \mathbf{ASD}^{\mu\nu;\alpha\beta}(q) \equiv \Pi^{\text{asd}}(q^2)\left(P_1^{\mu\nu;\alpha\beta} - P_2^{\mu\nu;\alpha\beta}\right), \quad (27)$$

$$(\hat{D}\mathbf{SD})^{\mu\nu;\alpha\beta}(q) = \mathbf{SD}^{\mu\nu;\alpha\beta}(q) \equiv \Pi^{\text{sd}}(q^2)\left(P_1^{\mu\nu;\alpha\beta} + P_2^{\mu\nu;\alpha\beta}\right). \quad (28)$$

The appearance of the **SD**-diagrams breaks the anti-duality of the two correlators Π and $\left(\hat{D}\Pi\right)$.

We can rewrite (26)–(28) to obtain the following representation for the OPE-induced part of the correlator:

$$\Pi_{\text{OPE}}^{\mu\nu;\alpha\beta}(q) = P_1^{\mu\nu;\alpha\beta}\left[\Pi^{\text{sd}}(q^2) + \Pi^{\text{asd}}(q^2)\right] + P_2^{\mu\nu;\alpha\beta}\left[\Pi^{\text{sd}}(q^2) - \Pi^{\text{asd}}(q^2)\right]. \quad (29)$$

As a simple consequence of this representation and (5) we have the useful relation

$$\Pi_{\text{OPE}}^{\mu\nu;\mu\nu}(q) = 6\Pi^{\text{sd}}(q^2). \quad (30)$$

Using (27) and (28), one can easily calculate the OPE coefficients for the different diagrams. For example, let us consider the $\langle\bar{q}Gq\rangle$-condensate and its contribution to the coefficient $b_\pm$. Indeed, we know that this contribution is of the **ASD**-type, that is

$$\Pi_{\langle\bar{q}Gq\rangle}^{\mu\nu;\alpha\beta} = c(q^2)\left(P_1^{\mu\nu;\alpha\beta} - P_2^{\mu\nu;\alpha\beta}\right).$$

Therefore, for a light-like vector z, one has

$$\Pi_{\langle\bar{q}Gq\rangle} = \Pi_{\langle\bar{q}Gq\rangle}^{\mu\nu;\alpha\beta} g_{\mu\alpha} z_\nu z_\beta = c(q^2)\frac{2(q\cdot z)^2}{q^2}.$$

This quantity reduces to the linear combination of $\langle\bar{q}Gq\rangle$-condensate contributions to the correlator for the vector currents (see [7,3]). In this way, we get the formula

$$\Pi_{\langle\bar{q}Gq\rangle} = \frac{-320(q\cdot z)^2}{81q^6}\pi\alpha_{\text{s}}\langle\bar{q}q\rangle^2$$

from which we then obtain the fraction 80/81 appearing in (14) and (15).

If $\mathbf{SD}_{\mu\nu;\alpha\beta}(q) = 0$, then we would have the same SRs for ρ- and b_1-mesons. We process this hypothetical SR within the standard approach without α_{s}-corrections in the perturbative contribution and obtain the following values for the low-energy parameters of a hypothetical ρb_1-meson in an anti-dual model of Nature:

$$m_{\rho b_1} = (0.865 \pm 0.030)\,\text{GeV}; \quad f_{\rho b_1} = (0.162 \pm 0.005)\,\text{GeV}; \quad s_{\rho b_1} = 1.58\,\text{GeV}^2. \quad (31)$$

We see that the mass and the coupling constant of the ρ-meson are not so much affected by this (anti-)duality breakdown (10% for the mass). The case of the b_1-meson is quite the opposite. Here the mass falls down to 45% (the constant to 16%, see (21)). This seems to be quite natural. In the case of the ρ-meson, the deformation of the SR is large (the quark condensate contribution is enhanced by a factor of 3.25), but its functional dependence on the Borel parameter M^2 is almost the same. This is not the case for the b_1-meson. The deformation of the SR due to the opposite sign of the quark condensate contribution is essential and this results in such a large effect for the mass of the b_1-meson. Thus, we can conclude that the origin of the differences in ρ- and b_1-meson masses and tensor couplings is the anti-duality breakdown in QCD due to the 4-quark condensate.

4 QCD vacuum tensor susceptibility

The QCD vacuum tensor susceptibility χ has been introduced in [5,6] in order to analyze, in the QCD SR approach, the nucleon tensor charges g_{T}^u and g_{T}^d. It is defined through the correlator (1) as

$$\chi = \frac{\Pi_\chi(0)}{6\langle\bar{q}q\rangle}, \quad \Pi\chi(q^2) \equiv \Pi^{\mu\nu;\mu\nu}(q^2). \quad (32)$$

He and Ji [5] obtained for $\Pi_\chi(0)$ the value

$$\frac{1}{12}\Pi_\chi(0) \approx 0.002\,\text{GeV}^2. \quad (33)$$

In order to obtain a reliable estimate in our approach we substitute the decomposition (2) in (32), use the relation (30) and arrive at the expression

$$\Pi_\chi(q^2) = 3\left(\Pi_+(q^2) + \Pi_-(q^2)\right) = 6\Pi^{\text{sd}}(q^2). \quad (34)$$

This relation clearly demonstrates that Π_χ is formed by the **SD** part of OPE, i.e., by the *four-quark condensate* contribution. Using the dispersion relations (6) we have

$$\frac{1}{12}\Pi_\chi(0) = \frac{1}{4\pi}\int_0^\infty \frac{\rho_+^{\text{phen}}(s) + \rho_-^{\text{phen}}(s)}{s}\text{d}s, \quad (35)$$

and using the phenomenological models for the spectral densities $\rho_\pm(s)$ in (8), the value of $\Pi_\chi(0)$ can be expressed in terms of the mesonic static characteristics (the analogous formula has been published in [14]):

$$\frac{1}{12}\Pi_\chi(0) = \frac{(f_{b_1}^{\text{T}})^2 - (f_\rho^{\text{T}})^2 - (f_{\rho'}^{\text{T}})^2}{2} + \frac{s_{\rho,\rho'} - s_{b_1}}{16\pi^2} = \begin{cases} -0.0055 \pm 0.0008\,\text{GeV}^2 & [\text{NLC}] \\ -0.0053 \pm 0.0021\,\text{GeV}^2 & [\text{Stand.}] \end{cases}, \quad (36)$$

presented in (18)–(19) for the NLC SR and (20)–(21) for the standard SR respectively[4]. Note here that both results are very close one to another due to strong cancellations in the difference (36). So, just this combination accumulates the effect of the four-quark part of the whole condensate contribution. If we return to the example of an anti-dual model of Nature (see the end of the previous section, (31))

[4] The depicted errors are obtained by a special invented χ^2-criterium and take into account only the SR stability

where this contribution is absent, we obtain the exact cancellation in (36), i.e. $\Pi_\chi(0) = 0$.

B&O in [2] have used the specific representation that leads to the decomposition

$$\Pi_\chi(q^2) = 12\Pi_1(q^2) + 6q^2\Pi_2(q^2), \tag{37}$$

where $\Pi_1(q^2) = (1/2)\Pi_+(q^2)$ and $q^2\Pi_2(q^2) = (1/2)\left(\Pi_-(q^2) - \Pi_+(q^2)\right)$. Erroneously suggesting that $\lim_{q^2\to 0}\left[q^2\Pi_2(q^2)\right] = 0$ and using the trick suggested in [15] based on the dispersion relation[5]

$$\Pi_\chi^{\text{n.p.}}(0) \equiv \frac{1}{12}\Pi_\chi(0) = \frac{1}{\pi}\int_0^\infty \frac{\rho^{\text{phen}}(s) - \rho^{\text{pert}}(s)}{s}\mathrm{d}s, \tag{38}$$

they concluded that

$$\Pi_\chi(0)^{\text{n.p.}} = \Pi_1^{\text{n.p.}}(0) = (f_{b_1}^{\mathrm{T}})^2 - \frac{s_{b_1}}{8\pi^2} \approx -0.008\,\mathrm{GeV}^2. \tag{39}$$

But we see from our analysis that the value of $(\Pi_-(q^2) - \Pi_+(q^2))$ is identically equal to 0 only in an absolutely self-dual world, which is definitely not realized in QCD:

$$\begin{aligned}\frac{\Pi_-(0) - \Pi_+(0)}{2} &= \frac{s_{\rho,\rho'} + s_{b_1}}{8\pi^2} - (f_\rho^{\mathrm{T}})^2 - (f_{\rho'}^{\mathrm{T}})^2 - (f_{b_1}^{\mathrm{T}})^2 \\ &= -0.0060 \pm 0.0017\,\mathrm{GeV}^2 \text{ NLC}. \end{aligned}\tag{40}$$

This value is comparable with the value of the B&O estimate (39) for $\Pi_1^{\text{n.p.}}(0)$ and should definitely be taken into account. Comparing the two estimates, our (36) and B&O (39), one sees a not so large deviation from one another. One should not be surprised because radiative corrections significantly reduce the B&O value to $\Pi_1^{\text{n.p.}}(0) \approx -0.003\,\mathrm{GeV}^2$. For this reason the actual magnitude of our total correction to this estimate is of the order of 100%. When our paper was finished we found the paper of [14], which contains an estimate of the correlator, $(1/12)\Pi_\chi(0) = -(0.0083 - 0.0104)\,\mathrm{GeV}^2$, using the constituent quark model. The authors of this paper have also determined a rather wide window for VTS by analogy of (36) using QCD SR results from different sources in the literature: $(1/12)\Pi_\chi(0) = -(0.0042 - 0.0104)\,\mathrm{GeV}^2$. As we pointed out in the Introduction, since these meson constants appear in VTS in the form of a difference, one has to define them more precisely and in the framework of a unified approach. So the large width of this window is not a surprise for us.

Finally, let us briefly discuss the effect of our estimate of the VTS on the nucleon tensor charges. Here we follow the pioneering paper by He and Ji [6] where these charges were roughly estimated using two types of SRs. Our result (36) increases the lower (decreases the upper) boundary for the g_{T}^u (g_{T}^d) charge approximately by a factor of 1.4:

$$g_{\mathrm{T}}^u = 1.47 \pm 0.76; \tag{41}$$

$$g_{\mathrm{T}}^d = 0.025 \pm 0.008. \tag{42}$$

(The results of He and Ji $g_{\mathrm{T}}^u = 1.33 \pm 0.53$ and $g_{\mathrm{T}}^d = 0.04 \pm 0.02$ have been obtained for, in our opinion, a value which is too low, being $\Lambda_{\mathrm{QCD}} = 100\,\mathrm{MeV}$. We, instead, use the value of $\Lambda_{\mathrm{QCD}} = 250\,\mathrm{MeV}$.)

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Appendix

A Expressions for non-local contributions to SR

The form of the contributions of the NLCs to OPE (11) depends on the model of the NLC. At the same time the final results of the SR processing demonstrate stability to the variations of the NLC model provided the scale of the average vacuum quark virtuality λ_q^2 is fixed. Here we use the model (delta-ansatz) suggested in [7] and used extensively in [3]; this model leads to Gaussian decay for the scalar quark condensate, $\langle\bar{q}(0)E(0,z)q(z)\rangle \sim \langle\bar{q}q\rangle \exp\left(-|z^2|\lambda_q^2/8\right)$ (see the details in [7]), dominated in $b_\pm$ via b_4. Here the factorization hypothesis is applied for the four-quark condensate, reducing its contribution to a pair of scalar condensates. In the NLC approach this may lead to an overestimate of the four-quark condensate contribution due to evident neglecting of the correlation between these scalar condensates, see Fig. 2.

In this model we obtain the "coefficients" for OPE in the SR (16) and (17),

$$\frac{b_\mp\left(M^2\right)}{\mp 2} = b_2\left(M^2\right) + b_3\left(M^2\right) \pm b_4\left(M^2\right), \tag{A.1}$$

where b_2 corresponds to the vector ($\langle\bar{q}\gamma_m q\rangle$), b_3 to the quark–gluon–quark ($\langle\bar{q}G_{\mu\nu}q\rangle$) and b_4 to the four-quark ($\langle\bar{q}q\bar{q}q\rangle$) vacuum condensate contributions (here $\Delta \equiv \lambda_q^2/(2M^2)$),

$$b_2\left(M^2\right) = -16; \tag{A.2}$$

$$b_3\left(M^2\right) = \frac{4\left(60 - 273\Delta + 359\Delta^2 - 134\Delta^3\right)}{3\left(1-\Delta\right)^3}; \tag{A.3}$$

$$\begin{aligned} b_4\left(M^2\right) &= 24\left(\Delta - 7\right)\frac{\log\left(1-\Delta\right)}{\Delta} \\ &+ 4\frac{25\Delta^2 - 21\Delta - 6}{\left(1-\Delta\right)^2}. \end{aligned}\tag{A.4}$$

The gluonic contribution $a_\pm$ coincides in this model with the standard expression (13). For quark and gluon condensates we use the standard estimates (for "renorm-in-

[5] Here $\rho^{\text{phen}}(s)$ and $\rho^{\text{pert}}(s)$ are the corresponding spectral densities; the difference of these functions validates the usage of the unsubtracted dispersion relation

variant" quantities in (A.5) we do not refer to any normalization point)

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = 1.2 \cdot 10^{-2}\,\mathrm{GeV}^4,$$
$$\langle \sqrt{\alpha_s} \bar{q} q \rangle^2 = 1.83 \cdot 10^{-4}\,\mathrm{GeV}^6, \qquad \text{(A.5)}$$
$$\lambda_q^2 \left(\mu^2 \approx 1\,\mathrm{GeV}^2\right) \equiv \frac{\langle \bar{q} \nabla^2 q \rangle}{\langle \bar{q} q \rangle} = \frac{\langle \bar{q}\,(\mathrm{i} g \sigma_{\mu\nu} G^{\mu\nu})\, q \rangle}{2\langle \bar{q} q \rangle} = 0.4 \pm 0.1\,\mathrm{GeV}^2. \qquad \text{(A.6)}$$

B Input parameters in the B&B and B&O papers

The groups of authors of [11] and of [2] used different definitions of the initial parameters for processing the SRs. Namely, B&O used the following set of values (without any indication on the scale at which renormalization non-invariant quantities are determined):

$$\alpha_s \approx 0.1\pi = 0.314, \quad 4\pi^2 \langle \bar{q} q \rangle = -0.55\,\mathrm{GeV}^3,$$
$$\langle \sqrt{\alpha_s} \bar{q} q \rangle^2 \approx 0.61 \cdot 10^{-4}\,\mathrm{GeV}^6, \qquad \text{(B.1)}$$

whereas B&B[6] (on the scale $\mu^2 \approx 1\,\mathrm{GeV}^2$)

$$\alpha_s = 0.56, \quad \langle \bar{q} q \rangle = (-0.250)^3\,\mathrm{GeV}^3,$$
$$\langle \sqrt{\alpha_s} \bar{q} q \rangle^2 \approx 1.37 \cdot 10^{-4}\,\mathrm{GeV}^6. \qquad \text{(B.2)}$$

This resulted in approximately the same overall contributions of the quark condensate in both papers, see the end of Sect. 2.

[6] Let us recall that the standard value is $\langle \alpha_s^{1/2} \bar{q} q \rangle^2 \approx 1.83 \cdot 10^{-4}\,\mathrm{GeV}^6$ [12]

References

1. J. Govaerts, L.J. Rubinstein, F. De Viron, J. Weyers, Nucl. Phys. B **283**, 706 (1987)
2. V.M. Belyaev, A. Oganesyan, Phys. Lett. B **395**, 307 (1997)
3. A.P. Bakulev, S.V. Mikhailov, Phys. Lett. B **436**, 351 (1998)
4. Particle Data Group Booklet, July 1998
5. Hanxim He, Xiangdong Ji, Phys. Rev. D **52**, 2960 (1995)
6. Hanxim He, Xiangdong Ji, Phys. Rev. D **54**, 6897 (1996)
7. S.V. Mikhailov, A.V. Radyushkin, Phys. Rev. D **45**, 1754 (1992); Sov. J. Nucl.Phys **49**, 494 (1989); A.P. Bakulev, A.V. Radyushkin, Phys. Lett. B **271**, 223 (1991)
8. V.M. Belyaev, B.L. Ioffe, ZhETF **83**, 876 (1982); A.A. Ovchinnikov, A.A. Pivovarov, Yad. Fiz. **48**, 1135 (1988); A.A. Pivovarov, Kratk. Soobshch. Fiz. (Bull. Lebedev Phys. Inst.) **5**, 3 (1991)
9. M. D'Elia, A. Di Giacomo, E. Meggiolaro, Phys. Rev. D **59**, 054503 (1999); E. Meggiolaro, hep-lat/9909068
10. H.G. Dosch, M. Eidemller, M. Jamin, E. Meggiolaro, hep-ph/0004040
11. Patricia Ball, V.M. Braun, Phys. Rev. D **54**, 2182 (1996)
12. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **147**, 385, 448 (1979)
13. D. Becirevic et al., hep-lat/9808187; hep-lat/9808129
14. Wojciech Broniowski, Maxim Polyakov, Hyun-Chul Kim, Klaus Goeke, Phys. Lett. B **438**, 242 (1998)
15. V.M. Belyaev, I.I. Kogan, Int. J. Mod. Phys. A **8**, 153 (1993)